INTERNATIONAL MATHEMATICS TOURNAMENT OF TOWNS

JUNIOR PAPER: YEARS 8,9,10

Tournament 42, Northern Autumn 2020 (O Level)

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Note: Each contestant is credited with the largest sum of points obtained for three problems.

1. Is it possible to choose 100 points on a circle so that there are exactly 1000 right-angled triangles whose vertices are among the chosen points? (3 points)

2. A group of 8 tennis players compete in one tournament per year using the Olympic system; that is, the players are randomly split into 4 pairs, the winners are then randomly split into 2 pairs that play in the semifinals and the winners of the semifinals play in the final. It turned out that after several years all players played each other exactly once. Prove that

   (a) each player played in the semifinals more than once. (2 points)

   (b) each player played in at least one final. (3 points)

3. There are n stones in a pile. Two players play a game taking turns. They can take either 1 stone from the pile or a prime number of stones which divides the current number of stones in the pile. The player who takes the last stone wins. For which n can the first player win for sure no matter how the other player plays? (5 points)

4. There is an equilateral triangle with side d and a point P such that the distances from P to the vertices of the triangle are positive numbers a, b and c. Prove that there exists a point Q and an equilateral triangle with side a such that the distances from Q to the vertices of this triangle are b, c and d. (5 points)

5. The director of a zoo has bought eight elephants numbered 1, 2, ..., 8. He has forgotten how much they weigh, but he remembers that each elephant, starting with elephant 3, has its weight equal to the sum of the weights of the two before it. One day there was a rumour that one of the elephants had lost some weight. How can the director make two weighings on a set of balance scales (using elephants only) to either find the elephant that had lost weight or conclude that the rumour was false? (It is known that no elephant gained weight and at most one elephant lost weight.) (5 points)